COSC 205 DISCRETE MATHEMATICS
SAMPLE QUESTIONS FOR EXAM I

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All questions on this examination will be graded. You have the entire class period to work on the exam.

· Answer all questions in the space provided on this paper.

· Show ALL your work on this paper.

The following symbols and notations are used in this exam:

~ logical NOT	$\lceil x \rceil$ ceiling of x
$\wedge$ logical AND	$\lfloor x \rfloor$ floor of x
$\vee$ logical OR	a|b a divides b
$\forall$ universal quantifier, for all	= equality
$\exists$ existential quantifier, there exists	<> inequality
=> single implication, if/then	$\therefore$ therefore, it follows that
<=> double implication, if and only if	

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1. Given boolean variables p and q, construct a truth table for the expression (~p $\wedge$ q).

p	q	~p	~p$\wedge$ q

2. Is the expression ~(~p $\wedge$ q) logically equivalent to the expression p $\vee$ ~q ? You MUST justify your answer in order to receive credit. (Use the truth table).

3. a) Rewrite the statement "No good cars are cheap" in the form "∀ x, if P(x) then ~Q(x).".

b) Indicate whether the following argument is valid or invalid, and justify your answers.

No good car is cheap.
<u>A Rimbaud is a good car</u>
∴ A Rimbaud is not cheap.

4. Let proposition P be " If you are in Goshen, then you are in U.S.A."

(a) State the contrapositive of P.

(b) State the converse of P.

5. What is the negation of the statement:

∀ students S, if S studies at Goshen College, then S goes on SST

6. Rewrite the statement "Everybody is good at some task" as a formal statement using quantifiers and variables.

7. What is the negation of the statement:

∃ students S, if S studies at Goshen College, then S is taking CS205

8. Rewrite the following statement in if-then form.

Being on time each day is necessary condition for keeping this job.

9. What is a tautology?

10. Given the following statements:

if Sue wears a red hat then Sue is not a member of "The Club"
Sue does not wear a headband
Joe lives on South Street or Joe lives on North Street
Sue lives on South Street or Sue lives on North Street
if Sue lives on South Street then Sue is a member of "The Club"
if Joe wears a headband then Joe lives on North Street
if Sue wears a headband, then Sue lives on North Street
Sue wears a red hat
Who lives on North Street (choose one answer)?
( )Sue
( )Joe
( )cannot be determined from these statements
( )these statements lead to a contradiction

11. Read the following proof:

PROPOSITION: ∀ integers a and b, if a and b are both prime and even then a = b.

Suppose there are integers a and b that are both prime and even. Also suppose that a <> b. Since a and b are even, a = 2r and b = 2s for some integers r and s. Consequently, 2|a, r|a, 2|b and s|b. Since a is prime, its only divisors are 1 and itself. Hence r=1 and a=2. Similarly, since b is prime its only divisors are 1 and itself, and so s=1 and b=2. It then follows that a=b. This conflicts with the statement that a<>b, hence the supposition is false and the proposition is true.

This is an example of (select one):

( ) proof by contraposition
( ) proof by contradiction
( ) proof by induction
( ) an invalid argument
( ) proof by direct argument

12. Prove the following statements that are TRUE and give A counter example to disprove those that are FALSE.

(a) The product of any two even integers is even.

(b) The difference of any two odd integers is even.

SELECTED THEOREMS

3.1.1 The sum of any two even integers is even.
3.2.1 Every integer is a rational number.
3.2.2 The sum of any two rational numbers is rational.
3.3.1 For all integers a, b, c, if a|b and b|c then a|c.
3.3.2 Any integer n > 1 is divisible by a prime number.
3.4.1 Given an integer n and positive integer d, there exist unique integers q and r such that n = d*q + r and 0 <= r < d.
3.4.2 Any integer is either even or odd.
3.4.3 Any two consecutive integers have opposite parity.
3.6.2 The sum of any rational number and any irrational number is irrational.
3.6.3 Given any integer n, if n2 is even, then n is even.
??? For any integer a and any prime number p, if p|a then p does not divide (a+1).