Homework: #2, Class #2 SOLUTIONS
Discrete Mathematics (Course Number: MTH-129-51)
Prof. G. Safko
Due: Class #3

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$p \wedge q \rightarrow \sim r$
$p \vee \sim q$
$\sim q \rightarrow p$
$\therefore \sim r$

Row	p	q	r	$p \wedge q$	$\sim r$	$\sim q$	$p \wedge q \rightarrow \sim r$	$p \vee \sim q$	$\sim q \rightarrow p$	$\sim r$
1	T	T	T	T	F	F	F	T	T	F
2	T	T	F	T	T	F	T	T	T	T
3	T	F	T	F	F	T	T	T	T	F
4	T	F	F	F	T	T	T	T	T	T
5	F	T	T	F	F	F	T	F	T	F
6	F	T	F	F	T	F	T	F	T	T
7	F	F	T	F	F	T	F	T	F	F
8	F	F	F	F	T	T	T	T	F	T

hypothesis conclusion

This is invalid. Rows 2,3,4 of the hypothesis premises are the only rows that are true, and the same rows in the conclusion are T,F,T. If the hypothesis premises were valid (true), then all three of the conclusions would have been valid (true), but the truth table reports "False" on row 3. Therefore the initial argument is invalid.

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$p \rightarrow r$
$q \rightarrow r$
$\therefore\ p \vee q \rightarrow r$

Row	p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	$p \vee q \rightarrow r$
1	T	T	T	T	T	T	T
2	T	T	F	T	F	F	F
3	T	F	T	T	T	T	T
4	T	F	F	T	F	T	F
5	F	T	T	T	T	T	T
6	F	T	F	T	T	F	F
7	F	F	T	F	T	T	T
8	F	F	F	F	T	T	T

hypothesis conclusion

This is valid. Rows 1,3,5,7,8 are true in both the hypothesis premises and the conclusion

#23
Prove or disprove (via truth tables) the following:

Oleg is a math major or Oleg is an economics major
If Oleg is a math major, then Oleg is required to take Math 362.
∴Oleg is an economics major or Oleg is not required to take Math 362.

Let M = Oleg is a math major
Let E = Oleg is a economics major
Let C = Oleg must take the Math 362 course

$M \vee E$
$M \rightarrow C$
$\therefore E \vee \sim C$

Row	M	E	C	~C	$M \vee E$	$M \rightarrow C$	$E \vee \sim C$
1	T	T	T	F	T	T	T
2	T	T	F	T	T	F	T
3	T	F	T	F	T	T	F
4	T	F	F	T	T	F	T
5	F	T	T	F	T	T	T
6	F	T	F	T	T	T	T
7	F	F	T	F	F	T	F
8	F	F	F	T	F	T	T

hypothesis conclusion

This is invalid. Rows 1,3,5,6 of the hypothesis premises are the only rows that are true. If the hypothesis premises were valid (true), then all four of the conclusions would have been valid (true), but the truth table reports “False” on row 3. Therefore the initial argument is invalid.

NOTE: We could create a wff (well formed formula) by stating
$(M \vee E) \wedge (M \rightarrow C) \rightarrow E \vee \sim C$, but we can derive no further information using the rules of inference, so a truth table is our only hope.

#40

(For #40, assume Socko is telling the truth. By the problem statement, all the men are lying except for one. Try to contradict Socko’s statement, find out who is telling the truth, who is lying and then determine the guilty criminal)

Suppose Socko is telling the truth. Then Fats is also telling the truth because if Lefty killed Sharky then Muscles didn’t kill Sharky. Therefore, two of the men were telling the truth, which contradicts the fact that all were lying except one. Therefore Socko is not telling the truth: Lefty did not kill Sharky. Hence Muscles is telling the truth and all the others are lying. It follows then that Fats is lying, so Muscles killed Sharky

In addition:

1) Prove using truth tables that p NAND q $\equiv \sim(p\wedge q)$

p	q	p NAND q	$p\wedge q$	$\sim(p\wedge q)$
T	T	F	T	F
T	F	T	F	T
F	T	T	F	T
F	F	T	F	T

Identical, so it's true

2) Prove using truth tables that p NOR q $\equiv \sim(p\vee q)$

p	q	p NOR q	$p\vee q$	$\sim(p\vee q)$
T	T	F	T	F
T	F	F	T	F
F	T	F	T	F
F	F	T	F	T

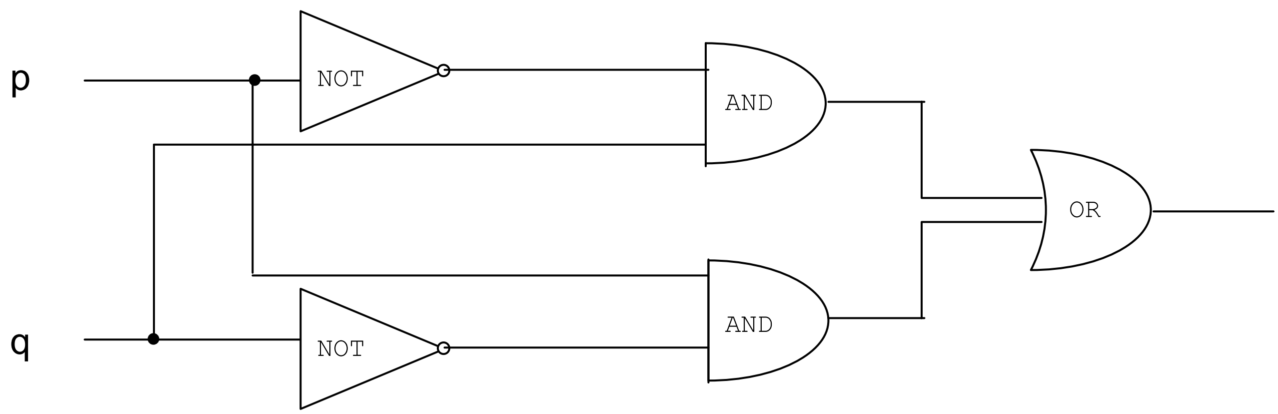
Identical, so it's true

3) Give a formal proof of the following tautology using the CP rule

$\sim(A \wedge B) \wedge (B \vee C) \wedge (C \rightarrow D) \rightarrow (A \rightarrow D)$

1. $\sim(A \wedge B)$ Premise
2. $(B \vee C)$ Premise
3. $(C \rightarrow D)$ Premise
4. $\sim A \vee \sim B$ 1, Tautology
5. A Premise (Start subproof of $A\rightarrow D$)
6. $\sim B$ 4,5, Elimination
7. C 2,6, Elimination
8. D 3,7,M.P.
9. $(A \rightarrow D)$ 5,8, CP (Finish subproof $A \rightarrow D$)
10. QED 1,2,3,9,CP

4) Given the following circuit:



a. Convert its input and outputs into a truth table

p	q	~p	~q	q ∧ ~p	p ∧ ~q	(q ∨ ~p) ∨ (p ∨ ~q)
T	T	F	F	F	F	F
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	F	F	F

b. Show that this circuit is exactly the circuit for XOR (Exclusive OR, denoted by ⊕)

p	q	p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F

The last column of this truth table is exactly the same as the last column in the previous table