

Homework: #3, Class #3 SOLUTIONS
Discrete Mathematics (Course Number: MTH-129-51)
Prof. G. Safko
Due: Class #4

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Let $R(m,n)$ be the predicate “If m is a factor of n^2 then m is a factor of n ”, with domain for both m and n being the set Z of integers.

- for $m = 25$ and $n = 10$, m is a factor of n^2 (100), but it is not a factor of 10
- One example: $m = 4$, $n = 6$
- for $m = 5$ and $n = 10$, m is a factor of n^2 (100), and it is also a factor of 10
- One example: $m = 2$, $n = 4$

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$\forall a \in Z$, $(a - 1)/a$ is not an integer. Proof by counterexample. Set $a = 1$, then the equation becomes 0, which is an integer

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\forall real numbers x and y , $(x + y)^{1/2} = x^{1/2} + y^{1/2}$. Set $x = y = 1$. Then $2^{1/2} \neq 1^{1/2} + 1^{1/2}$

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- There is a shape that is a rectangle and a square. (This is true)
- There is a shape that is a rectangle and not a square. (This is true)
- All squares are rectangles (This is true)

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The negation of “All dogs are loyal” is either “There is a dog who is not loyal” or “Some dogs are not loyal”.

The answers are c, f

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“There are no simple solutions to life’s problems”

The negation is: “There are simple solutions to life’s problems”

Using notation,

$\forall x \in \{\text{Life's problems}\} \text{ s.t. } \sim \text{SolutionsTo}(x)$

Where $\text{SolutionsTo}(x) = \text{true}$ if problem x has a simple solution

In addition:

Convert this quote from Abraham Lincoln into predicate calculus statements:

You can fool some of the people all of the time, all of the people some of the time, but you can't fool all of the people all of the time.

Hint: Use the following sets

$P = \{\text{set of all people}\}$

$T = \{\text{set of all possible time intervals}\}$

$F(p,t)$ = Returns "true" if person p is fooled in time element t

(One possible solution:)

$P = \{\text{set of all people}\}$

$T = \{\text{set of all time elements}\}$

$F(p,t)$ = returns "true" if person p is fooled in time element t

You can fool some of the people all of the time:

$\forall t \in T, \exists p \in P \text{ s.t. } F(p,t)$

all of the people some of the time:

$\forall p \in P, \exists t \in T \text{ s.t. } F(p,t)$

but you can't fool all of the people all of the time.:

$\exists t \in T, \exists p \in P, \sim F(p,t)$

Combined, you get:

$(\forall t \in T, \exists p \in P \text{ s.t. } F(p,t)) \wedge (\forall p \in P, \exists t \in T \text{ s.t. } F(p,t)) \wedge (\exists t \in T, \exists p \in P, \sim F(p,t))$