Homework: #4, Class #4 SOLUTIONS
Discrete Mathematics (Course Number: MTH-129-51)
Prof. G. Safko
Due: Class #5

Prove the following by direct proof, or disprove by proof by contradiction:

$\forall\ x \in \{\text{Irrationals}\}, x^2 \in \{\text{Irrationals}\}$

Invalid: Set $x = \sqrt{7}$. $x^2 = 7$, and $7 \notin \{\text{Irrationals}\}$

$\forall\ x \in \{\text{Negative Real Numbers}\}, x^2 > x$

Valid. All squares are positive, and x is always negative

$\exists\ x \in \{\text{Primes}\}, \exists\ y \in \{\text{Fibonacci Numbers}\},\ x = y$

Valid for $x = y = 3$

$\exists\ x,y \in \{\text{Primes}\}, \exists\ n \in \{Z^{\text{odd}}\},\ n = xy$ and $x \neq y$

Valid for $x = 3, y = 7$

Write your own predicates P(x), and Q(x) than can satisfy the following:

$\forall$ x, if P(x), then Q(x)

P(x) = is a positive number
Q(x) = is greater than 0

$\forall$ x, if ~Q(x), then ~P(x)

P(x) = is an odd number
Q(x) = is not divisible by 2

$\exists$ x, if P(x), then Q(x)

P(x) = is an odd number
Q(x) = is a prime number

$\exists$ x, if ~Q(x), then ~P(x)

P(x) = is an even number
Q(x) = is a Fibonacci number