Homework: #5 Solutions
Discrete Mathematics (Course Number: MTH-129-51)
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Page 139: #6

For example, let a = 1, b = 0. Then $(a + b)^{1/2} = (1)^{1/2} = 1$ and $(a)^{1/2} + (b)^{1/2} = (1)^{1/2} + (0)^{1/2} = 1$ also. Therefore $(a + b)^{1/2} = (a)^{1/2} + (b)^{1/2}$ for these values of a and b

Page 139: #8

Note: This solution was based on the second edition, where it asked to find a real number x such that $2^x > x^{10}$.
Second Edition Solution:
We need to find x such that $2^x \geq x^{10}$. By examination, x = 0 or x = 1 does the trick. In fact, any x such that $0 \leq x \leq 1$ will work. If the problem had asked for x > 1, then you would need to solve for x. Using logs, we would see need to solve $\log(2^x) \geq \log(x^{10})$., which is $x \log(2) \geq 10 \log(x)$. Regrouping, we get $x / (\log(x)) \geq 10/\log(2)$. Solving by trial and error comes up with $x \geq 60$ for $x \in \mathbf{Z}$
Third Edition Solution: Interestingly, I answered it in the above solution.

Page 141: #46

Suppose the n and m are integers such that n – m is even. To show that $n^3 - m^3$ is even we see that $n^3 - m^3 = (n - m)(n^2 + nm + m^2)$ (by algebra). Since $n^2 + nm + m^2$ is an integer (by previous theorems), when we multiply it by an even number (in this case n – m), we get an even number. Hence $n^3 - m^3$ is even.

Page 141: #47

Prove by showing a counterexample: Let n = 2. Then, n is prime but $(-1)^n = (-1)^2 = 1 \neq 1$

Page 141: #48

Prove by showing a counterexample: Let n = 3. Then, $m^2 - 4 = 9 - 4 = 5$, which is not composite Another way to look at this is to examine that $m^2 - 4$ is $(m - 2)(m + 2)$. Since a composite is a number r * s with neither r nor s = 1, then we can contradict this by setting m – 2 = 1, with m = 3.

Page 141: #49

Prove by showing a counterexample: Let n = 11. Then, $n^2 - n + 11 = 11^2 - 11 + 11 = 11^2$, which is not a prime number.

Also:

Proof via direct proof:

Prove that $n * (n+1)(n+2)(n+3) = x^2 - 1$ for $n,x \in \mathbf{Z}$

By algebra, it equals $n^4 + 6n^3 + 11n^2 + 5n = x^2$
Or
$n^4 + 6n^3 + 11n^2 + 5n + 1 = x^2$

which factors to
$(n^2 + 3n + 1)(n^2 + 3n + 1) = x^2$

which is a perfect square

Also:

If Floor(x) = $\lfloor x \rfloor$ = n where $n \in \mathbf{Z}$ such that $n \leq x < n + 1$
Is it also true for $x - 1 < n \leq x$?

You do not need to prove this, just show that it is so.

Use as an example, 2.9, 3.0, and 3.1

Floor(2.9) = 2
Therefore, n = 2
And it is true that $(2.9) - 1 < 2 \leq 2.9$

Floor(3.0) = 3
Therefore, n = 3
And it is true that $(3) - 1 < 3 \leq 3$

Floor(3.1) = 3
Therefore, n = 3
And it is true that $(3.1) - 1 < 3 \leq 3$

If Ceiling (x) = $\lceil x \rceil$ = n where $n \in \mathbf{Z}$ such that $n - 1 < x \leq n$
Is it also true for $x \leq n < x + 1$?

Use as an example, 2.9, 3.0, and 3.1

Ceiling(2.9) = 2
Therefore, n = 3
And it is true that $2.9 \leq 3 < (2.9 + 1)$

Ceiling(3.0) = 3
Therefore, n = 3
And it is true that $3.0 \leq 3 < (3.0 + 1)$

Ceiling(3.1) = 4
Therefore, n = 4
And it is true that $3.1 \leq 4 < (3.1 + 1)$