Review 1
Discrete Mathematics
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Name:
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~ logical NOT	$\forall$ universal quantifier, for all	$\Leftrightarrow$ double implication, if and only if
$\wedge$ logical AND	$\exists$ existential quantifier, there exists	= equality
$\vee$ logical OR	$\Rightarrow$ single implication, if/then	$\neq$ inequality

$\therefore$ therefore, it follows that

The Rules of Inference

Modus Ponens: $P \rightarrow Q$ P $\therefore Q$	**Modus Tollens:** $P \rightarrow Q$ $\sim Q$ $\therefore \sim P$	**Rule of contradiction** $\sim p \rightarrow c$ $\therefore p$
Generalization (Disjunctive Addition) p $\therefore p \vee q$ -or- q $\therefore p \vee q$	**Elimination (Disjunctive Syllogism)** $p \vee q$ $\sim q$ $\therefore p$ -or- $p \vee q$ $\sim p$ $\therefore q$	**Transitivity (Hypothetical syllogism)** $p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$
Specialization (Conjunctive Simplification) $p \wedge q$ $\therefore p$ -or- $p \wedge q$ $\therefore q$	**Conjunction (Conjunctive Addition)** p q $\therefore p \wedge q$	**Proof by division into cases** $p \vee q$ $p \rightarrow r$ $q \rightarrow r$ $\therefore r$

SELECTED THEOREMS AND LEMMAS

The sum of any two even integers is even.
Every integer is a rational number.
The sum of any two rational numbers is rational.
For all integers a, b, c, if $a|b$ and $b|c$ then $a|c$.
Any integer $n > 1$ is divisible by a prime number.
Given an integer n and positive integer d, there exist unique integers q and r such that $n = d*q + r$ and $0 \leq r < d$.
If n is a nonnegative integer and d is a positive integer, and if $q = \lfloor n / d \rfloor$ and $r = n - d\lfloor n / d \rfloor$ then $n = dq + r$ for $0 \leq r < d$
Any integer is either even or odd.
Any two consecutive integers have opposite parity.
The sum of any rational number and any irrational number is irrational.
Given any integer n, if n^2 is even, then n is even.

For any integer a and any prime number p, if p|a then p does not divide (a+1).
The product of n consecutive integers is divisible by n
Any integer can be factored into a series of prime numbers of integer powers
The square of an odd integer can be represented in the form 8m + 1 for some $m \in \mathbf{Z}$
There is no greatest integer
There is no integer that is both even and odd
The sum of a rational and an irrational number is irrational
The set of primes is infinite

Lemmas:

The product of any two even integers is even
The product of any two odd numbers is odd
The sum of an even and an odd is odd

REVIEW QUESTIONS

1. Rewrite the following quantified statement in English:

$\forall x \in D, \exists y \in D$ s.t. $xy = 1$

2. Rewrite the following English statement into a quantified statement

For a given x from the set of real numbers, there exists a y such that x times y is an odd number

3. Rewrite the following English statement into a quantified statement

For a given x from the set of real numbers, there exists a single unique y such that x times y equals 5

4. Given the set T = {the set of all irrational numbers}

Prove or disprove the following (justify your answer):

$\forall x,y \in T, x + y \in T$

$\forall x,y \in T, x - y \in T$

$\exists x,y \in T, x * y \in T$

$\exists x,y \in T, x / y \in T$

5. Fermat's Last Theorem states that there is no $n,x,y,z \in \mathbf{Z}$ for $n > 2$ that satisfies the equation

$$x^n + y^n = z^n$$

Suppose the restriction for n was changed to $n \geq 2$. Can the equation be satisfied? Prove or disprove your answer.

6. Give formal proofs for the following using the rules of inference for a conditional proof

a. $A \rightarrow (\sim B \rightarrow (A \wedge \sim B))$
b. $(A \vee B \rightarrow C) \wedge A \rightarrow C$
c. $(A \vee B \rightarrow C \wedge D) \rightarrow (B \rightarrow C)$

7. Write the following statement using predicate notation

If a person doesn't drink and the person has a car, then that person can be a designated driver

8. Write the contrapositive of #7

9. Write the converse of #7

10. Write the inverse of #7

11. If you are creating a truth table based on the following proposition:

$(p \vee q) \wedge (\sim r \wedge p)$

a. How many rows of a truth table would you need and why?
b. How many unique columns would you need (i.e., you would need unique columns, for p, q, and r, and for $p \vee q$, etc.)
c. Construct the truth table for this statement
d. Convert this truth table into a circuit using circuit symbols and connections

12. Prove or disprove the following:

a. For all even numbers x, y, and $y \neq 0$: x/y is an integer
b. For all even number x, y, and $y \neq 0$: x mod y = 0
c. For all rational numbers r and irrational numbers s: r * s is irrational
d. Prove that the square root of π is irrational (you can assume that π is known to be irrational)
e. Show that if n is an integer, then $n^3 - n$ is even
f. Show that for every positive integer n, $n^4 - n^2$ is divisible by 6 (Hint: Is it divisible by 2? Is it also divisible by 3?)

Definitions:
What is a tautology?
What is a contradiction?
What is a predicate?
What does "vacuously true" mean?